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Jayanta Kumar Ghosh: A Short Description of the Evolution of His Research Work

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1. Introduction

Professor Jayanta Kumar Ghosh was a legendary statistician. In 60 years of his research career, he worked in various areas of statistics including both the frequentist as well as the Bayesian paradigms. Prof. Ghosh's areas of interest included (but, not restricted to) classical statistical inference, sequential analysis, higher order asymptotics, rates of convergence, reliability theory, stochastic modelling, Bayesian asymptotics in parametric, non-parametric and semi-parametric inference, multiple decision theory, survey sampling, model selection, *etc.* He had applied his vast statistical knowledge to solve many real-life problems in geology, ecology, public health and environment studies. In his own words, *"many of the papers in the above areas solve long standing open problems or pioneer new areas, and have been cited often."* [1]

In his research career, Prof. Ghosh was honored with several awards including the Shanti Swarup Bhatnagar Award for Science and Technology in 1981, the Mahalanobis Gold Medal from Indian Science Congress Association in 1998, the P. V. Sukhatme Prize for Statistics in 2000, the Doctor of Science (D.Sc.) award by B. C. Roy Agricultural University, India in 2006, a Lifetime Achievement Award from the International Indian Statistical Association in 2010, and Padma Shri by the Government of India in 2014. He was selected as the 'President' of Statistics Section of the Indian Science Congress Association in 1991, and of the International Statistical Institute in 1993.

2. Early Career

Prof. Ghosh's research career began in early 1960s, while pursuing his Ph.D. dissertation under the supervision of Prof. Hari Kinkar Nandi. In his first two papers, he investigated the properties of Wald's sequential probability ratio test (SPRT) (Ghosh, 1960a,b). Prof. Ghosh continued to work on SPRT during 1960s and later in phases of his research life as well. Some pioneering contributions include formalization of the result of Stein establishing relationship between sufficiency and invariance (Hall *et al.* (1965)), the Ghosh-Pratt identity (Ghosh, 1961), *etc.*

[1]The quotations used in this manuscript are obtained from Prof. Ghosh's personal webpage: https://www.stat.purdue.edu/∼ghosh/.

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3. A Brief Description of Prof. Ghosh's Work in Classical Asymptotics

In Prof. Ghosh's words, "*my main work after this (SPRT) and till 1982 had been on higher order asymptotics*". His entire volume of work on classical asymptotics has been classified into seven broad categories in Clarke and Ghosal (2008). Here, we highlight only a few among them.

(i) *Bahadur representation of quantiles:* Prof. Ghosh relaxed the assumptions of Bahadur representation, which is a representation of the sample quantile as a sum of independent and identically distributed (i.i.d.) random variables, at the cost of a different ordered (in probability) remainder. The generalization to place in two aspects: first, from the existence of second order bounded derivative to the existence of first order (positive) derivative of the cumulative distribution function (c.d.f.), and second, from fixed dimension p to variable dimension $p_n = p + O\left(n^{-1/2}\right)$ (see Ghosh (1971)).

(ii) *Higher order asymptotics:* We split this topic into three sub-parts.

Edgeworth expansions. In his seminal work, Prof. Ghosh (with Prof. Rabi Bhattacharya) formalized the validity of r-th order Edgeworth expansion for smooth functionals of sample averages under appropriate moment conditions and some conditions on the associated characteristic functions (see Bhattacharya and Ghosh (1978, 1980)). Prof. Ghosh, along with his collaborators, also derived Edgeworth expansions for likelihood ratio type test statistics, which have non-normal limiting distributions (see, e.g., Chandra and Ghosh (1979)).

Second order efficiency and admissibility. Related to Edgeworth expansion are the second order efficiency and the second order admissibility type properties of estimators. The performance of asymptotically efficient estimators can be compared in the light of second order efficiency. Similarly, second order admissibility compares statistics using the second order risk. Along with his collaborators, Prof. Ghosh established pioneering results in comparing maximum likelihood estimator (MLE) and related estimators in terms of second order efficiency and admissibility (see, e.g., Ghosh and Subramanyam (1974); DasGupta and Ghosh (1983)).

Comparison of likelihood ratio, Wald and Rao's Tests. Comparison of the likelihood ratio test (LRT), Rao's and Wald's tests has been an important problem, which has drawn attention of many researchers in 1970s. Prof. Ghosh (along with Prof. Tapas Kumar Chandra) rigorously derived the asymptotic expansions of the distribution functions of LRT, Rao's and Wald's tests (see Chandra and Ghosh (1980)).

(iii) *Bartlett's correction.* A series of Prof. Ghosh's work hinges on Bartlett's correction. Asymptotic normality of LRT statistics with Bartlett's corrected error was proved for multidimensional settings in the seminal paper of Bickel and Ghosh (1990). This result was proved through a Bayesian argument, where Bartlett's approximation was applied to the limiting posterior distribution. Further studies in this route related to frequentist and Bayesian Bartlett's approximation can be obtained from Ghosh and Mukerjee (1991, 1992a).

(iv) *Neyman-Scott problem:* In the Neyman-Scott problem, one is interested in estimating the common parameter θ based on the sequence $\{X_n\}_{n\geq 1}$ of independent random variables with X_i having the density $f(\cdot, \theta, \xi_i)$. Prof. Ghosh (along with Prof. Bhanja) proposed asymptotically efficient estimators of θ under parametric and semiparametric setup depending on the nature of the sequence $\{\xi_n\}_{n\geq 1}$, considering it to be fixed or random following a common distribution. This work and related extensions can be found in Bhanja and Ghosh (1992a,b,c).

4. A Brief Description of Prof. Ghosh's Work on Bayesian Inference

In the later part of his research career, Prof. Ghosh contributed equally, if not more to Bayesian asymptotics. In his own views, his work in Bayesian asymptotics can be classified into four broad categories:

(i) *"The first related to theorems like posterior normality, necessary and sufficient conditions for a normalized posterior to converge to a non-degenerate distribution, etc."* Prof. Ghosh extended the Bernstein von-Mises (BvM) theorem, which shows asymptotic normality of the posterior distribution after proper centering and scaling, in various aspects. Extensions of the BvM theorem and related work include providing precise conditions for higher order expansion of the posterior distribution under the marginal distribution of the data (Ghosh *et al.*, 1982); providing a version of the BvM theorem where the posterior is calculated given the sample mean instead of the full sample (Clarke and Ghosh, 1995); proving posterior strong consistency under weak assumptions on the normalizing factors and for non-regular densities (Ghosh *et al.*, 1994; Ghosal *et al.*, 1995); obtaining first-order asymptotic approximation to the posterior distribution of the unknown change point θ in a change point problem (Ghosal *et al.*, 1999c), *etc.*

(ii) *"In the second set of problems I* (Prof. Ghosh) *try to derive priors for which posterior probabilities are close to frequentist probabilities in various senses. This is relevant for validating non-informative priors or constructing confidence intervals."* In this context, Prof. Ghosh introduced the idea of probability matching prior, where a prior is so chosen that the posterior joint cumulative distribution function (c.d.f.), or the highest posterior density regions of a standardized version of the parametric vector matches with the corresponding frequentist c.d.f. up to an order of $o(n^{-1/2})$ (Ghosh and Mukerjee, 1993b), or $o(n^{-1})$ (Ghosh and Mukerjee, 1993a; Datta and Ghosh, 1995). Other related objective priors like reference priors have been studied in Ghosh and Mukerjee (1992b), and a comparison of several objective priors has been done in Datta and Ghosh (1995).

(iii) *"In the third set of problems I* (Prof. Ghosh) *deal with Bayesian analysis of infinite dimensional problems like Bayesian survival analysis, Bayesian density estimation, Bayesian semiparametric,* etc. *A major concern here is the consistency of posterior and rate of convergence."* Prof. Ghosh made significant contribution in the area of Bayesian nonparametric, and has been one of the main architects in designing the modern asymptotic theory of Bayesian nonparametric. His work in Bayesian nonparametric started with the work Ghosh and Ramamoorthi (1995), co-authored with Prof. R. V. Ramamoorthi, where they studied the convergence of the posterior distribution towards the true underlying distribution in the context of survival data both in censored and uncensored cases (see also Ghosh *et al.* (1999)). Generally, the choice of an appropriate prior is crucial towards establishing posterior consistency for infinite-dimensional models. Prof. Ghosh has shown posterior consistency for the parameter of symmetry for any unknown symmetric distributions under a semiparametric setup using symmetrized Pòlya tree prior (Ghosal *et al.*, 1999a). Similarly, Pòlya tree priors and Dirichlet mixtures of a normal kernel were used in a regression context in Amewou-Atisso *et al.* (2003).

The above mentioned paper as well as the modern Bayesian asymptotic theory for infinite-dimensional models uses Schwartz's theorem (Schwartz, 1965), which is *the right tool for studying consistency* in infinite-dimensional problems. Implementation of the Schwartz's idea using a sieve based approach was used for density estimation by Dirichlet mixture of normal densities in Ghosal *et al.* (1999b), and with the logistic Gaussian process prior in Tokdar and Ghosh (2007).

Consistency is just the first step. Given consistency, one would be interested in results on rates of convergence, which has also been investigated in (Ghosal *et al.*, 2000).

(iv) Finally, Prof. Ghosh has *"made major progress in understanding Bayesian and Empirical Bayes model selection rules in high dimensional problems. I* (Prof. Ghosh) *have also been working on Bayes Testing, Model Selection in low dimensional problems and have thrown light on some recent as well as long standing asymptotic problems."* The early work on Prof. Ghosh on model selection dates back to 1975, when Ghosh and Subramanyam (1975) had formalized the idea of separated hypothesis in terms of the L_1 distance. Among the more recent works, generalizations of BIC for high-dimensional data are considered in Berger *et al.* (2003); Chakrabarti and Ghosh (2006). A pertinent review of application of model selection procedures like AIC, BIC and their comparison is provided in Chakrabarti and Ghosh (2011). In Mukhopadhyay and Ghosh (2003), the relative performance of several parametric empirical Bayes methods was compared with AIC and BIC using asymptotic results and simulation studies both under the $0-1$ as well as the prediction loss.

In Bayesian model selection (or, hypothesis testing), difficulties arise when improper non-informative priors are used to calculate the Bayes factors. Several methods have been proposed to remove these difficulties. Ghosh and Samanta (2002) discusses a unified derivation of some of these methods. Towards the implementation of Bayes factor in factor model, Dutta and Ghosh (2013) provides justification of the regularity conditions needed for path sampling.

In the context of multiple testing, Bogdan *et al.* (2007) compared the empirical Bayes approach with Benjamini-Hochberg method, focusing mainly on the 'sparse mixture' case (see also Bogdan *et al.* (2008)). Within a Bayesian decision theoretic framework, some asymptotic optimality properties of a large class of multiple testing rules were investigated in Bogdan *et al.* (2011). A comprehensive review of model selection and multiple testing can be obtained from Dutta *et al.* (2012).

5. Conclusion

Prof. Ghosh was equally interested in meaningful applications of statistics in scientific and industrial research (Sinha and Sinha (2017)). For example, Maiti *et al.* (2016) discusses discrepancies in data sources for estimating the household consumption expenditure to derive Gross Domestic Product (GDP) of India and provides simple implementable strategies to improve the estimation of GDP.

Apart from this, there are multiple research areas in which Prof. Ghosh has contributed significantly and yielded numerous elegant and insightful research articles which could not be covered in this short write up. An account of his personal and research careers can be obtained from Ramamoorthi (2018), and also from Dasgupta (2017).

6. Jayanta Kumar Ghosh Endowment Lecture

In the J. K. Ghosh endowment lecture, I presented my recent research work on category learning (see Mukhopadhyay *et al.* (2021)). An abstract of this work is given below:

Understanding how adult humans learn to categorize can shed novel insights into the mechanisms underlying experience-dependent brain plasticity. Drift-diffusion processes are popular in such contexts for their ability to mimic underlying neural mechanisms but require data on both category responses and associated response times for inference. Category response accuracies are, however, often the only reliable measure recorded by behavioral scientists to describe human learning. Building carefully on drift-diffusion models with latent response times, a novel biologically interpretable class of 'inverse-probit' categorical probability models is derived for such data. The model, however, presents significant identifiability and inference challenges. These challenges are addressed via a novel projection-based approach with a symmetry preserving identifiability constraint that allows working with conjugate priors in an unconstrained space. This model is adapted for group and individual level inference in longitudinal settings. Building again on the model's latent variable representation, an efficient Markov chain Monte Carlo algorithm is designed for posterior computation. The method's empirical performances are evaluated through simulation experiments. The method's practical efficacy is illustrated in applications to longitudinal tone learning studies.

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